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TWO STUDIES ON ZODIACAL LIGHT

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by V. G. Fesenkov

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TWO STUDIES ON ZODIACAL LIGHT

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1. The Conditions of Disintegration of Asteroids According to the Observed Peculiarities of Zodiacal Light

[This is a translation of an article written by V. G. Fesenkov in Izvestiya Astrofiz. Inst. Akademiya Nauk Kazakh SSR (News of the Astrophysical Institute, Academy of Sciences Kazakh SSR), Vol VIII, 1959, pages 3-11.]

Recent data (Bibl. 1, 2, 3) establish that zodiacal light characterizes the scattering of light conducted by dust particles in interplanetary space without any notable admixture of free electrons. In effect, all the observed peculiarities of zodiacal light -- form and position relative to the ecliptic, absolute brightness, color, polarization, and even the probable relationship to the solar corona -- can be fully explained by the dust nature of zodiacal light, i.e., by the scatter of solar light by fine particles. The presence in the solar system of the gas and even the electron components does not manifest itself photometrically in any way. However, fine dust cannot long maintain itself in space because of the retarding effect of solar radiation and the corpuscular emission of the Sun. As a result of the former cause alone, the entire amount of dust matter should precipitate onto the sun within the comparatively short time of the order of 100,000 years. Thus, the composition of the matter of zodiacal light is bound continually to renew itself through the perpetual influx of matter from the outside, and this extremely intensive process is a consequence of the gradual fragmentation of asteroids. This process is also accompanied by the release of meteorites, as is attested by the small cosmic ages of such bodies determined on the basis of their content of the helium isotope He_3 (Bibl. 4, 5).

Proceeding from the distribution of asteroids according to the angles of inclination of their orbits, it is possible to compute theoretically the isophotes of zodiacal light and to compare them with actual observations. This entails known difficulties, because the visible isophotes

of zodiacal light are influenced by various attendant components of night airglow of atmospheric and cosmic origin and by zodiacal twilight, i.e., by the illumination of the parts of zodiacal light close to the horizon by other and much more brighter parts of the same light located directly below the horizon and, lastly, by the additional scatter of the glow of zodiacal light caused by the troposphere.

My calculations (Bibl. 6) indicate that the effect of zodiacal twilight is not large and cannot considerably distort the observed isophotes of this phenomenon, but the illumination of the troposphere by that light is very appreciable because of its extensive angular dimensions; moreover, it is azimuthally irregular.

This results not only in a definite increase in the brightness of this phenomenon but also in a substantial expansion of the isophotes characterizing it. It is also possible that other causes contribute to this effect. However, all such extraneous influences prove to be much smaller at a vertical orientation of the axis of zodiacal light relative to the horizon, which takes place, e.g., below the circles of the tropics toward the equinox. We have conducted numerous observations of zodiacal light in the Libyan Desert south of Aswan in October and November 1957, when it was possible within a single night to observe this phenomenon normal to the horizon in the east before the sunrise and at the same time considerably inclined in the west soon after the onset of night. In this connection it was possible to obtain a more graphic proof of the relationship between the visible distortion of the isophotes and the angle of inclination of the ecliptic to the horizon, which can to a major extent be explained by the above-indicated causes.

However, even for the normal position of zodiacal light, we find that its true isophotes are comparatively wide even though fully symmetrical relative to the ecliptic.

These isophotes, as obtained from observations conducted in Egypt, are illustrated in Fig. 1.

On the other hand, the theoretical isophotes of zodiacal light can be computed on assuming a definite law of the density distribution of dust matter in interplanetary space as a function of distance from the Sun, a definite type

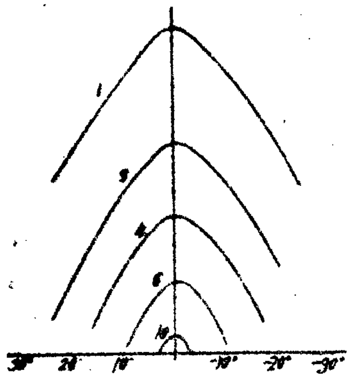

Fig. 1

of the scattering function, and the postulate that dust matter ensues from the disintegration of asteroids without any appreciable initial relative velocity.

The density distribution of dust matter can be most logically obtained from the condition of its stationariness and its inverse ratio to distance from the sun. To calculate the disintegration of asteroids it is necessary to proceed from the known distribution according to the angles of inclination of their orbits relative to the plane of the ecliptic (Bibl. 7). The scattering functions of dust matter in interplanetary space are unknown, but it can be assumed that the result is hardly influenced by their nature.

In effect, let us take three radically different forms of scattering functions: the simply spherical one, for which the calculations are the simplest; the function typical for the Earth's atmosphere, i.e., reflecting the properties of the Rayleighian and aerosolic scattering of light with an asymmetry amounting to 5.6, represented by the expression

$$f(\vartheta) = 1 + 5{,}5\,(e^{-2\vartheta} - 0{,}009) + 0{,}55\cos^2\vartheta,$$

and the purely aerosolic scattering function with a very large asymmetry amounting to 12, obtained from the preceding function by a suitable exclusion of Rayleigh's scatter.

$$f_1(\vartheta) = 1 + 11{,}1\,(e^{-3\vartheta} - 0{,}009).$$

The results of the calculation of the corresponding isophotes of zodiacal light for the above three scattering functions are illustrated in Figs. 2, 3, and 4.

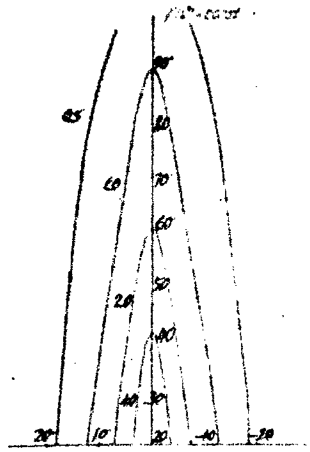

Fig. 2.

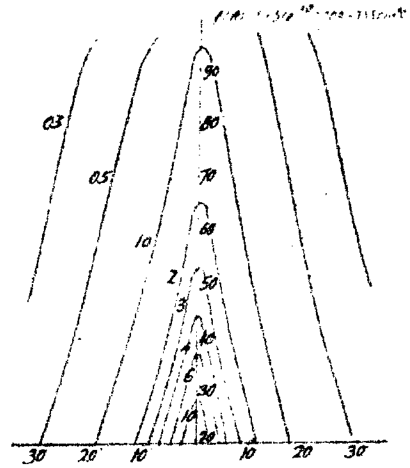

Fig. 3.

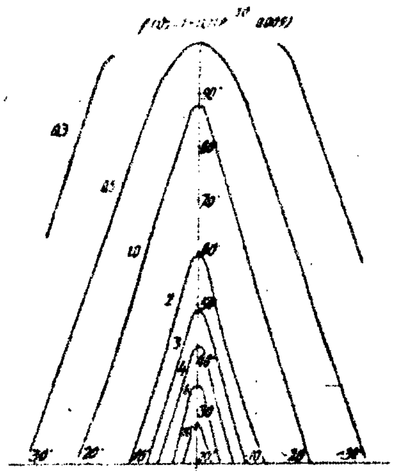

Fig. 4.

As can be seen from the diagrams, the spherical scattering function provides the most compressed isophotes of zodiacal light, but these isophotes expand -- not much, to be sure -- relative to the ecliptic when the asymmetry increases. In a like manner, the brightness of distribution in zodiacal light in the plane of the ecliptic proves to be nearly identical for the last two cases, as is illustrated in Fig. 5.

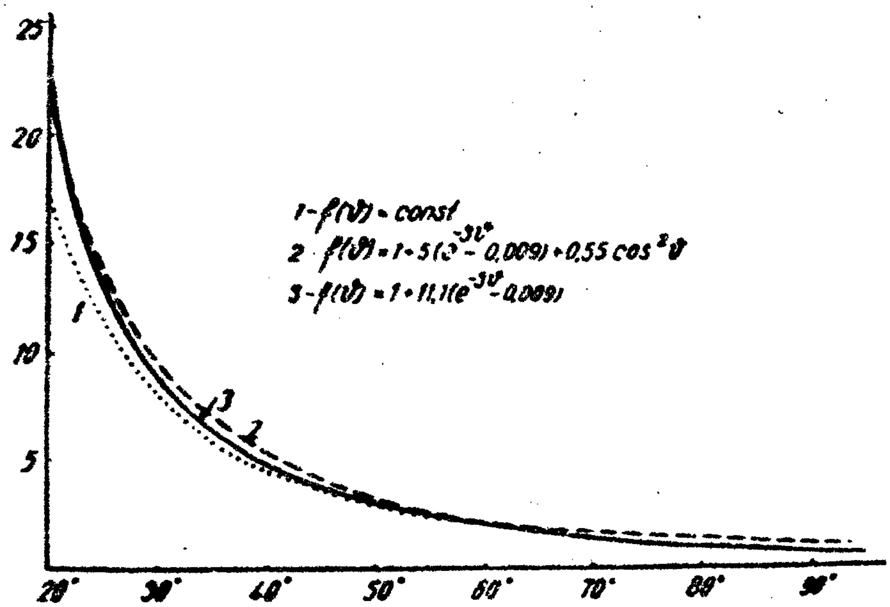

Fig. 5.

Thus the nature of the isophotes of zodiacal light is little affected by the type of scattering function, and it depends distinctly on the distribution of the orbits of dust particles according to their angles of inclination relative to the ecliptic.

It can be stated that the theoretically computed isophotes of zodiacal light prove to be, all other conditions being equal, more compressed relative to the ecliptic, if they are computed on the basis of the known distribution of the inclinations of asteroidal orbits.

To characterize the isophotes of zodiacal light, it is possible to represent them approximately in the form of an isosceles triangle normal to the horizon, and to determine the ratio of its base length to its height. For the actually observed isophotes deduced after introducing the necessary corrections adopted at present, we obtain, on the average, 1.3 in (southern) Egypt, while for the theoretically computed isophotes, even for the widest ones with the maximum value of asymmetry, we obtain only 0.56.

Thus there exists a notable divergence in form between the computed and the actually observed isophotes,

which apparently can be explained only by the conditions of formation of zodiacal light, i.e., from our standpoint, by conditions accompanied by a gradual disintegration of asteroids into dust.

We shall proceed from the hypothesis that the gradual disintegration of asteroids can be likened to the spewing of particles of matter with identical velocity uniformly in all directions. If this velocity accounts for a major part of the orbital velocity of an asteroid, then a sufficient propagation of dust particles into space to both sides of the ecliptic should occur and, consequently, a corresponding widening of the isophotes of zodiacal light should occur. As I had shown previously (Bibl. 7), these isophotes can be represented by the following simple expressions:

$$I_{zl} = \int_0^\infty \frac{e^{-k_1 \frac{\Delta}{\gamma} \sin\beta} f(\vartheta)\, d\Delta}{r}, \qquad (1)$$

where r is the distance from the sun
Δ is the distance from the observer
β is the geocentric latitude
k_1 is a coefficient determing the nature of the relationship between the spatial distribution of dust particles and the heliocentric angle relative to the plane of the ecliptic:

$$\Phi(\varphi) \sim \frac{e^{-k_1 \sin\varphi}}{r}.$$

On determining from (1) that coefficient, it is possible subsequently to use the integral equation of form

$$\Phi(\varphi) = \int_\varphi^{\frac{\pi}{2}} \frac{F(i)\, di}{\sqrt{\sin^2 i - \sin^2\varphi}}$$

in order to find the distribution function of dust particles F(i) according to the angle of inclination of their orbits i. That function is directly related to

the relative velocity of the spewing forth of particles during the asteroidal disintegration.

It is, however, simpler to proceed backward and initially to establish the relationship between the relative velocity of the spewing forth of dust particles and the distribution of their orbits according to angles of inclination i. On designating by v the orbital velocity of an asteroid and by w the relative velocity of the spewing forth of a dust particle from its surface, let us compose the conventional expressions for areal integrals:

$$y\frac{dx}{dt} - x\frac{dy}{dt} = C\cos i;$$

$$z\frac{dy}{dt} - y\frac{dz}{dt} = C\sin\Omega\sin i;$$

$$x\frac{dz}{dt} - z\frac{dx}{dt} = C\cos\Omega\sin i;$$

in which, for the asteroid under examination, it can be assumed that x = r and y = z = 0 (the y-axis is oriented toward the side of orbital motion). Obviously, we obtain

$$\cos(x, w) = \sin\vartheta\cos\lambda;$$

$$\cos(y, w) = \cos\vartheta;$$

$$\cos(z, w) = \sin\vartheta\sin\lambda,$$

if and represent the polar coordinates determining the direction of the spewed-forth particle, in which connection is analogous to angular distance calculated from the direction toward the apex of the movement of the asteroid, and is analogous to longitude calculated from the plane of its orbit.

In addition, we obviously obtain

$$\frac{dx}{dt} = w\cos(x, w);$$

$$\frac{dy}{dt} = w\cos(y, w) + v;$$

$$\frac{dz}{dt} = w\cos(z, w).$$

Whence we find that

$$\operatorname{tg} i = -\frac{K \sin\vartheta \sin\lambda}{1 + K\cos\vartheta}, \tag{2}$$

in which connection

$$K = \frac{w}{v}$$

is relative velocity of the ejection of the dust particle, expressed in parts of orbital velocity.

Generally speaking, the probability of the ejection of matter from the surface of an asteroid constitutes a $f(\lambda, \vartheta)$ -function of ϑ, λ and K. The number of dust particles, whose orbits differ in angle of inclination i $\div$ i + di, can be represented by the relation

$$P \sim \int f(\vartheta, \lambda) \frac{\sin\vartheta d\vartheta d\lambda}{dt}.$$

In the event of uniform spewing forth in all directions, $f(\vartheta, \lambda) = \text{const}$, in which connection λ and ϑ are interrelated by formula (2). Let us determine on a representative sphere the curved line corresponding to a constant value of the angle of inclination (i = const); $\lambda = \varphi(i, \cos\vartheta)$.

In this case our integral can be thus represented

$$P(i) = \int \frac{d\varphi(i, \cos\vartheta)}{di} \sin\vartheta d\vartheta$$

or thus

$$\frac{d}{di} \int_{\vartheta_1}^{\vartheta_2} \arcsin\left[\frac{\operatorname{tg} i (1 + K\cos\vartheta)}{K \sin\vartheta}\right] \sin\vartheta d\vartheta,$$

where ϑ_2 and ϑ_1 are the upper and lower limits, respectively, corresponding to the natural values of the angle λ.

The integral represented by the above expression can be easily computed by the numerical method. We did this for two different values of the K-constant, to wit: K = 0.1 and K = 1.0.

The computing procedure is reduced to constructing

the curves of $\lambda=\varphi(i, \cos\vartheta)$. The values of λ in degrees are plotted along the axis of ordinates, and the values of $\cos\vartheta$ from - 1 to +1 are plotted along the axis of abscissas. Subsequently, the trapezoid method is used to determine the areas enclosed in between these curves and the axes of ordinates corresponding to i = 0°, and the differences in the areas of the corresponding adjacent curves are taken. Subsequently these differences are divided by the corresponding interval in the angle of inclination. Lastly, let us note that the maximum possible value of i, equal to

$$max\, i = \text{arc tg} \frac{K}{\sqrt{1-K^2}},$$

in the first case amounts to (K = 0.1) i_{max} = 5.75°, and in the second, to i_{max} = 90°.

The results of calculations, expressed in arbitrary units, are cited in Table 1.

Table 1

$\cos\vartheta$	0,998	0,99	0,98	0,95	0,90	0,80	0,70	0,60	0,50	0,40	0,30	0,20
Δi	19,6	24,4	18,2	35,8	39,5	53,5	38,0	29,7	22,9	20,2	15,8	11,4
i	0,16	0,53	0,88	1,34	1,96	2,74	3,05	4,06	4,50	4,86	5,16	5,40
$\frac{\Delta P}{\Delta i}$	59,2	50,0	50,4	48,3	55,0	52,2	54,5	49,2	50,6	50,2	56,2	55,7

The figures in this Table are not distinguished by a high accuracy, because the evaluation of the lower limit of $\cos\vartheta$ was conducted in an approximate manner only. Nevertheless, it becomes completely clear that the distribution function of orbits of ejected particles proves to be constant throughout the area in which motion is possible.

In an analogous manner we compute the distribution function of the orbits according to the angles of inclination to the ecliptic in the case of K = 1, when relative velocity of ejection equals orbital velocity. The results are presented in Table 2.

Table 2

$tg\,i$	0,1	0,2	0,3	0,4	0,5	0,6	0,7
$\cos\vartheta$	0,980	0,923	0,835	0,724	0,600	0,474	0,343
Δi	5,72	5,60	5,38	5,10	4,77	4,40	4,03
$\bar{i}$	2,8	8,5	14,0	19,2	24,2	28,8	33,0
$\frac{\Delta P}{\Delta i}$	32,3	30,6	30,4	29,0	29,5	26,9	24,7

$tg\,i$	0,8	1,0	1,2	1,6	2,0	4,0	
$\cos\vartheta$	0,118	0,000	−0,181	−0,428	−0,600	−0,882	−1,000
Δi	3,66	6,34	5,20	7,80	5,43	12,53	14,04
$\bar{i}$	36,8	41,8	47,6	54,1	60,7	69,7	83,0
$\frac{\Delta P}{\Delta i}$	25,3	24,5	21,2	17,9	16,8	10,3	3,9

As can be seen from Table 2, this distribution function decreases gradually though slowly with increasing angle of inclination. In its smoothed-out form it can be thus presented.

i	0°	5	10	15	20	25	30	35	40°
$\varphi(i)$	1,0	0,998	0,993	0.978	0,952	0,917	0,865	0,823	0,765
i	45°	50	55	60	65	70	75	80	90°
$\varphi(i)$	0,710	0,646	0,587	0,522	6,442	0,339	0,258	0,174	0

The next step is to compute the effect of these functions on the distribution, according to angles of inclination, of the over-all whole of dust matter in the solar system. Let us assume, e.g., that the distribution of the orbits of asteroids according to angles of inclination such as is cited by the known catalogs, is represented by the f(i)-function which is expressed by a curve whose ordinates decrease with increasing i. Let us consider that every point on that curve is blurred in accordance with the nature of $\varphi(i)$, i.e., either turns into a homogeneous spot if $\varphi(i)$ is constant or into an inhomogeneous spot if that function decreases with increasing i.

Generally speaking, the obtained total effect which could be expected for every value of i is determined by the integral

$$N(i) = c\int f(\xi)\,\varphi(\xi - i)\,d\xi. \qquad (3)$$

if $\varphi(i)$ is constant, so that $\varphi(i) = \text{const}$, if $-a \leqslant i \leqslant a$ and $\varphi(i) = 0$, if $|i| \geqslant a$, then .

<u>Table 2</u>

$\operatorname{tg} i$	0,1	0,2	0,3	0,4	0,5	0,6	0,7
$\cos\vartheta$	0,980	0,923	0,835	0,724	0,600	0,474	0,343
Δi	5,72	5,60	5,38	5,10	4,77	4,40	4,03
$\bar{i}$	2,8	8,5	14,0	19,2	24,2	28,8	33,0
$\frac{\Delta P}{\Delta i}$	32,3	30,6	30,4	29,0	29,5	26,9	24,7
$\operatorname{tg} i$	0,8	1,0	1,2	1,6	2,0	4,0	
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The next step is to compute the effect of these functions on the distribution, according to angles of inclination, of the over-all whole of dust matter in the solar system. Let us assume, e.g., that the distribution of the orbits of asteroids according to angles of inclination such as is cited by the known catalogs, is represented by the f(i)-function which is expressed by a curve whose ordinates decrease with increasing i. Let us consider that every point on that curve is blurred in accordance with the nature of $\varphi(i)$, i.e., either turns into a homogeneous spot if $\varphi(i)$ is constant or into an inhomogeneous spot if that function decreases with increasing i.

Generally speaking, the obtained total effect which could be expected for every value of i is determined by the integral

$$N(i) = c\int f(\xi)\,\varphi(\xi - i)\,d\xi. \tag{3}$$

if $\varphi(i)$ is constant, so that $\varphi(i) = \text{const}$, if $-a \leqslant i \leqslant a$ and $\varphi(i) = 0$, if $|i| \geqslant a$, then .

$$N(i) = \int_{i-a}^{i+a} f(\xi)\, d\xi.$$

We adopt as f(i) the distribution of asteroids according to the inclinations of their orbits as given in, e.g., my work (Bibl. 8), which in its smoothed-out form is represented by the following relative curve:

i	0	5	10	15	20	25	30	35
$f(i)$	1,00	0,977	0,872	0,537	0,248	0,091	0,021	0,0062

On calculating the integral in the above two cases, we find the following relative total distribution of inclinations for the over-all whole of the orbits of the dust particles generated by asteroids:

$K = 0{,}1$

i	0	5	10	15	20	25	30	35°
$N(i)$	1,00	0,967	0,827	0,542	0,264	0,111	0,037	0,0087

$K = 1$

i	0	10	20	30	40	50	60°
$N(i)$	1,00	0,986	0,943	0,866	0,767	0,647	0,506

Upon obtaining this distribution according to the inclinations of the dust particles (Fig. 6) generated by asteroids in accordance with the afore-mentioned scheme, it is finally possible to compute the corresponding theoretical isophotes of zodiacal light and to compare them with the actually observed ones. The results obtained are fairly obvious. When K = 0.1, no significant difference from the isophotes obtainable for the case of K = 0 manifests itself. The theoretical form of zodiacal light is represented by a nearly identically compressed set of isophotes. Conversely, when K = 1, the computed isophotes of zodiacal light should prove to be considerably rarefied.

Thus, the observations can evidently be represented by some intermediate value equaling, in all probability, 0.4 or 0.5 for the relative velocity of ejection as expressed in units of orbital velocity of the asteroid. Inasmuch as the orbital velocity of asteroids at their mean distance from the sun approximates 20 km/sec,

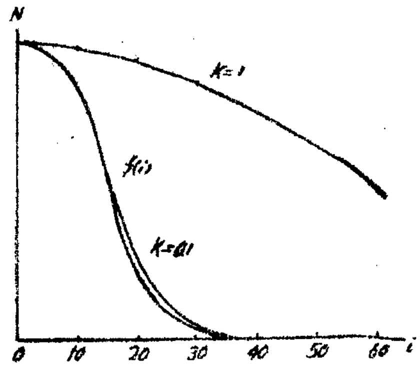

Fig. 6.

therefore the necessary velocity of the spewing forth of the dust complementing the matter of zodiacal light should reach a fairly high value: of the order of five to 10 km/sec.

A more rigorous approach to the soving of this problem could be the following:

from the approximate identity

$$e^{-k_1 (\sin\varphi_1 - \sin\varphi_2)} \int_0^1 \frac{dx e^{-k \arccos (x \cos\varphi_1)}}{\sqrt{1-x^2}} : \int_0^1 \frac{dx e^{-k \arccos (x \cos\varphi_2)}}{\sqrt{1-x^2}},$$

where $x = \frac{\cos i}{\cos\varphi}$,

it is possible to establish the ratio K to K_1. Thus, e.g., if K = 0.2, we find that $K_1 = 9$, and if $K = 0.1$, then K_1 analogously becomes one half as high.

The coefficient K characterizes the distribution f(i) of the orbits of asteroids according to their angles of inclination to the ecliptic, while K_1 enters directly into the formula determining the isophotes of zodiacal light.

However, there is no immediate need for such fairly ponderous calculations. The above-formulated purely qualitative conclusion as to the velocity of the outward flight of dust particles from the asteroids is a quite obvious one. However, such considerable velocities of

ejection of particles [when they occur] as a result of collisions between asteroids appear to be unfeasible, because the relative velocity itself of the asteroids is quite low as they move in one and the same direction.

Thus, the observed width of the isophotes of zodiacal light should apparently be thus explained: the dust matter forming this light is generated not only by asteroids but also by periodic comets, which are characterized by a much broader distribution of orbits according to angle of inclination to the ecliptic. A different alternative may be that in addition to the comparatively large asteroids known to us there also exists a multitude of other, smaller asteroids with a broader range of the distribution of orbits according to angle of inclination. At any rate, the observed form of zodiacal light attests that it originates from a complex of bodies that are sufficiently widely distributed relative to the ecliptic but bear no relation whatsoever to the solar equator. A clearer characterization of that complex would definitely require the prior determination of the brightness of the zodiacal component at the pole of the ecliptic, on excluding all other components. This should constitute the next task in the studies of zodiacal light.

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2. On Zodiacal Twilight

[This is a translation of an article written by V. G. Fesenkov in Izvestiya Astrofiz. Inst. Akademiya Nauk Kazakh SSR (News of the Astrophysical Institute, Academy of Sciences, Kazakh SSR), Vol VIII, 1959, pages 13-18.]

The problem of the reduction of the photometric observations of zodiacal twilight cannot as yet be considered ultimately resolved. Hitherto it was deemed necessary to take into account the purely atmospheric component of night airglow, which superimposes itself on zodiacal light, and to consider as well the corresponding scatter of light in the troposphere and the galactic component represented by the integral light of stars and their light scattered in interstellar space. The first component, which depends mainly on the state of the ionosphere, is a variable one and it should be kept continually under control. Nonetheless, a comparison of the isophotes of zodiacal light obtained at various times during its different inclinations to the horizon indicates that these reductions are insufficient. In effect, it becomes clear that the form of the isophotes is the more compressed relative to the ecliptic, the wider the angle composed by the ecliptic and the horizon. Thus, e.g., in Aswan (Egypt), where during pre-dawn hours in autumn the ecliptic is oriented completely normal to the horizon, the isophotes of zodiacal light are, after the introduction of corrections for the above-mentioned components, undoubtedly narrower than the analogous isophotes obtained by the same methods in our Central-Asian deserts.

Thus, there should exist other components of night airglow which have not yet been taken into consideration. Primarily, one such additional componet is the illumination of the troposphere by the zodiacal light itself, which should exert a major effect on the distribution of brightness in the troposphere. Of fundamental influence here are the extensive angular dimensions of this phenomenon. The

brightness of the daytime sky is insignificant in relation to the intensity of the solar disk, solely because the latter has very small angular dimensions. If, however, it is represented that the sun is extended in the form of a uniform disk throughout the entire celestial firmament, then the brightness of the sky resulting from the produced atmospheric scatter will, even at the zenith, amount to approximately 10-12 percent of the brightness of the solar disk itself.

It was shown previously (Bibl. 1) that in the case of a somewhat idealized zodiacal light represented by a set of isophotes normal to the horizon, and for the points of the circle of altitudes corresponding to a zenith distance of 80°, the additional glow, at normal atmospheric transparency, amounts to approximately six percent for the axis of the zodiacal light and is as great as 30 percent at the points of that light spaced 40 degrees apart azimuthally from that axis.

Thus, this effect, which is particularly pronounced at an inclined position of the ecliptic, should undoubtedly appreciably widen the observed isophotes of zodiacal light. An accurate consideration of this effect consitutes a quite complex problem, and it can be achieved only with the aid of sufficiently detailed auxiliary tables. The numerical compilation of such tables should be our next task.

In the present article we will examine the analogous problem of the relative effect on the brightness of zodiacal light observed at the altitude of, e.g., 10 degrees above the horizon by the incomparably brighter parts of that light located directly below the horizon -- the so-called zodiacal twilight. To obtain a qualitative solution of the problem, let us examine the following example. Let the ecliptic be inclined to the horizon at an angle of 60 degrees, and the sun lie below the horizon at an angle of 18 degrees. The true isophotes of zodiacal light, such as can be plotted outside the terrestrial atmosphere and in the absence of the galactics, are expressed by the formula

$$
I = \int_0^\infty \frac{f(\vartheta)\, e^{-9\frac{\Delta \sin\beta}{r}}\, d\Delta}{r^3},
$$

derived from the postulate of the gradual disintegration of asteroids (Bibl. 2) under the condition of stationariness.

Here Δ is the linear distance, from the observer, of a certain parcel of interplanetary matter,

r is the corresponding distance from the Sun,

β is the ecliptical latitude, and

$f(\vartheta)$ is the scattering function characterizing the interplanetary medium.

Adopting the standard scattering function derived by Ye. V. Pyaskovskaya-Fesenkina for the Earth's atmosphere (3), to wit:

$$
f(\vartheta) = 1 + 5\,(e^{-3\vartheta} - 0{,}009) + 0{,}55\cos^2\vartheta,
$$

we obtain the following values of brightness in various points of zodiacal light, according to which the following set of isophotes could be constructed (Table 1).

Table 1

β \ $l - l_0$	5°	10°	20°	30°	45°	60°	90°	120°	150°	180°
0°	322	90,5	21,1	9,24	3,96	2,21	1,11	0,82	0,75	0,73
5	11,1	8,74	5,45	3,49	1,98	1,28	0,76	0,62	0,59	0,59
10	3,55	3,11	2,29	1,70	1,13	0,82	0,55	0,48	0,48	0,47
20	1,11	1,04	0,88	0,72	0,54	0,43	0,34	0,32	0,34	0,35
30	0,56	0,54	0,49	0,42	0,34	0,29	0,24	0,24	0,27	0,28
40	0,35	0,34	0,32	0,30	0,26	0,22	0,19	0,20	0,22	0,23
60	0,20	0,19	0,19	0,18	0,17	0,16	0,15	0,16	0,16	0,17
80	0,15	0,15	0,15	0,15	0,14	0,14	0,14	0,14	0,14	0,14

The errors of the values cited in Table 1 amount to approximately 1-3 percent.

Table 2 cites analogous values of the brightness of zodiacal light computed according to the same formula but for the purely aerosolic scattering function:

$$
f(\vartheta) = 1 + 11{,}1\,(e^{-3\vartheta} - 0{,}009),
$$

with an asymmetry equaling 12. The calculations are executed with a high accuracy, but they differ little from the previous ones. As can be seen, the isophotes of zodiacal light depend only slightly on the asymmetry of the scattering function.

<u>Table 2</u>

β \ $l-l_0$	5°	10°	20°	30°	45°	60°	90°	120°	150°	180°
0	331	95,6	21,4	8,7	3,6	2,0	0,92	0,60	0,48	0,45
5	16,2	11,6	6,1	3,6	1,92	1,20	0,65	0,46	0,38	0,36
10	5,36	4,41	2,82	1,88	1,14	0,79	0,48	0,36	0,31	0,30
20	1,57	1,45	1,10	0,84	0,57	0,43	0,31	0,25	0,23	0,22
30	0,71	0,67	0,59	0,48	0,37	0,30	0,23	0,20	0,18	0,18
40	0,41	0,40	0,36	0,32	0,27	0,23	0,19	0,17	0,16	0,16
60	0,20	0,20	0.20	0,19	0,18	0,17	0,15	0,14	0,14	0,14
80	0,15	0,15	0,15	0,15	0,14	0,14	0,14	0,14	0,13	0,13

The figures in Table 2 are expressed in units of brightness, viz.: in the number of fifth-magnitude stars per square degree, which approximately corresponds with reality.

Every parcel of zodiacal light outside the atmosphere can be regarded as a cosmical light source producing the illumination of the entire atmosphere and, in particular, the phenomonon of twilight. We should find the integral effect produced by all such parcels taken together.

Considering that the color index of zodiacal light differs in no way from that of solar light, it is possible to employ the relative set of the isophotes of twilight glow such as are detected by direct observations of ordinary twilight. It is possible, of course, to employ the twilight theory developed by various investigators (Bibl. 4, 5), but that theory does not adequately consider the higher-order light scatter, which is particularly notable at considerable distances from the horizon and outside the vertical of the Sun. Hence, it is more preferable to utilize direct observations of the brightness of twilight, e.g., those conducted for various dips of the Sun below the horizon (Bibl. 6) and for various azimuths relative to the solar vertical. Table 3 cites the corresponding values of twilight brightness for a given point in the

sky at the height of 10 degrees above the horizon. On the basis of such data it is possible to construct a complete set of isophotes characterizing this phenomenon.

Table 3

$h_0 = 10°$

h \ A	0°	22,5°	45°	90°	135°
0°	150	90.0	58	34	35
−1°	74,0	50,0	35,4	20,9	19,1
−2°	34,7	26,3	19,1	11,8	8,7
−3°	15,0	13,0	10,0	6,0	3,0
−4°	5,8	5,0	3,8	1,9	1,0
−5°	1,90	1,82	1,26	0,63	0,26
−6°	0,70	0,65	0,35	0,09	0,07
−7°	0,24	0,20	0,08	0,026	0,023
−8°	0,083	0,073	0,036	0,010	0,008
−9°	0,030	0,025	0,015	0,0045	0,0030

We can also obtain analogous tables for other circles of altitudes above the observer's horizon.

The brightness of zodiacal twilight can be represented by the expression:

$$\iint L(\beta, l - l_0) f(h, h_0, A_0 - A) \cos h\, dh\, dA,$$

in which the integration is over all altitudes and azimuths.

Here the function $L(\beta, l - l_0)$ represents the brightness of a parcel of zodiacal light located above the horizon, as taken from Table 1 or from the corresponding graphs of isophotes according to the coordinates β and $l - l_0$.

The function $f(h, h_0, A_0 - A)$ characterizes the twilight brightness dependent on the extent of the plunge of the Sun below the horizon h, the height h_0 of a given point of the sky above the horizon (in the given case 10°), and the difference $A_0 - A$ in the azimuths between the given sky point and the parcel of zodiacal light constituting a light source analogous to the Sun.

To facilitate the integration it is expedient to employ the graphic method. Assuming, as noted before,

that the inclination of the ecliptic to the horizon amounts to 60 degrees, and assuming the dip of the Sun below the horizon at the end of ordinary twilight to amount to 18 degrees, it is possible to convert all the ecliptical coordinates expressing the isophotes of zodiacal light (cf. Table 2) into the corresponding azimuthal coordinates and to outline in the latter coordinates the zodiacal light basically below the horizon and partly above it (Fig. 1). In addition, on a separate sheet of tracing paper, we plot in the same azimuthal coordinates a set of twilight isophotes for every position of the Sun below the horizon. To carry out the integration we superimpose the one graph on the other, so that their horizon lines would coincide; also, we cause the axis of symmetry of the twilight isophotes to pass through the selected parcel of zodiacal light representing a light source analogous to the Sun. In this case, the sought twilight effect at the examined sky point ($h_o = 10°$) will be proportional to the product of the brightness of that parcel of zodiacal light and the brightness of the sky determinable by the twilight isophotes taken, of course, for a corresponding dip of the illuminating parcel.

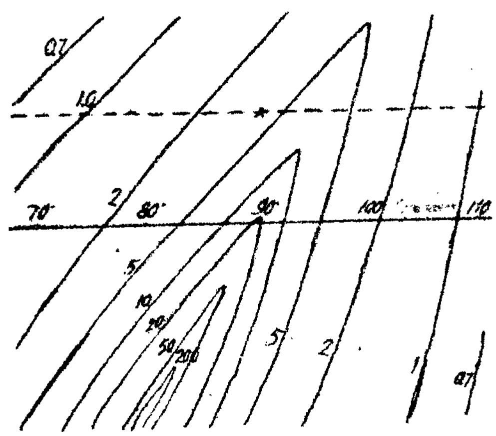

Fig. 1.

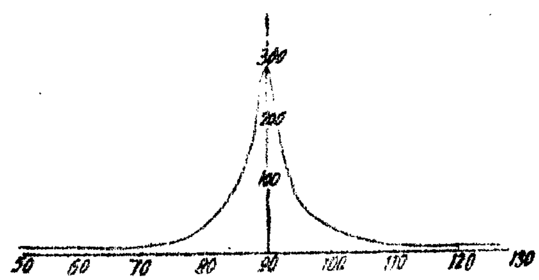

Fig. 2.

Thus, e. g., for an azimuth of 90°, for a sky point of 10° corresponding approximately to the axis of zodiacal light, and for the position of the illuminating parcel on the horizon itself, upon shifting the axis of symmetry of the twilight isophotes through every degree of the azimuth

to both sides of the sky point, we find the product of the afore-mentioned functions, which can be represented graphically in the form of a curve (Fig. 2). The area delimited by that curve represents the twilight effect which is produced by the cross section of zodiacal light along the entire horizon. We conduct an analogous calculation for all other cross sections of zodiacal light, taken at ever-increasing distances below the horizon through every altitude degree until the lowest point of their influence is attained below the horizin. The results of these calculations are cited in Table 4 for $h = 10°$.

Table 4

h \ A	40	60	70	80	90	100	110	130
0°	132	190	228	276	300	270	214	144
−1°	86	115	136	158	166	150	126	91
−2°	52	67	76	85	88	81	71	54
−3°	29	36	40	42	43	41	38	30
−4°	11,9	15,2	16,8	17,8	17,9	17,2	15,7	12,3
−5°	4,4	5.9	6,3	6,5	6,5	6,4	6,0	4,5
−6°	1,3	2,2	2,5	2,6	2,6	2,5	2,2	1,4
−7°	0,4	0,7	0,8	0,9	0,9	0,8	0,7	0,4
−8°	0,2	0,3	0,3	0,3	0,3	0,3	0,3	0,2
−9°	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
	250	335	390	448	470	430	365	265

The last row of figures in Table 4 cites the results of integration for each azimuth as executed by the trapezoid method representing a value proportional to zodiacal twilight, i.e., the additional illumination produced from below the horizon by the entire phenomenon of zodiacal light: such an additional illumination at the 10-degree altitude is illustrated for various azimuths in Fig. 3. As can be seen, it is represented by a very broad curve, considerably broader than the cross section of zodiacal light through its axis. If the absolute value of that additional illumination is considerable, then the visible isophotes of zodiacal light will prove to be appreciably widened. There results should be expresed in absolute units. For this purpose, it is possible to utilize the observations of twilight conducted by N. B. Divari in 1955

at the Montane Astrophysical Observatory of the Academy of Sciences Kazakh SSR by means of an electrophotometer in the solar vertical and in various portions of the spectrum. He cites figures on the brightness of twilight in terms of the number of fifth-magnitude stars per square degree.

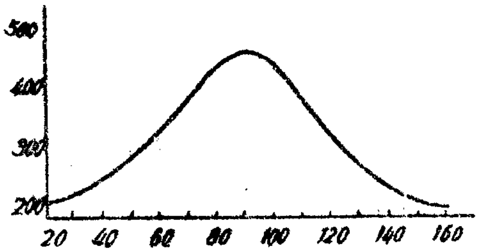

Fig. 3.

Primarily, it can be ascertained that these measurements coincide, as to the course of the twilight curve, quite satisfactorily with the preceding ones used as the basis for our determinations. A particularly fine coincidence can be observed for the two twilight curves constructed for the solar vertical for the point 10 degrees high above the horizon, commencing from the dip of the Sun below the horizon to two degrees and lower, as illustrated in Fig. 4. Consequently, to standardize our results, we assume, on the basis of Divari's observations, that the brightness of ordinary twilight at the point 10 degrees above the horizon on the solar vertical and during a three-degree dip of the Sun below the horizon amounts to, in the green rays of the spectrum, $9.0 \cdot 10^5$ in the above-mentioned units. The brightness of the light source, i.e., the Sun itself, in the same units amounts to $4 \cdot 10^{12}$.

Had the brightness of the Sun been equal to that of a single star of the fifth magnitude, it would have produced a twilight of a brightness amounting to only $9/4 \cdot 10^{-7}$. However, taking the correct values of the brightness of zodiacal light in accordance with the computed isophotes of that light, we assumed the twilight brightness to amount to 15 according to our nominal units, for the 10-degree high point above the horizon and for the three-degree dip of the Sun. Consequently, the obtained result, in order to be expressed in terms of the number of fifth-magnitude stars per square degree, has to be

multiplied by a factor equal to $9/4 \cdot 10^{-7}$: 15, i.e., $1.5 \cdot 10^{-8}$. This signifies that even at the brightest point of zodiacal twilight the intensity would be only $5 \cdot 10^{4} \times 1.5 \cdot 10^{-8}$, i.e., of the order of 10^{-3} one thousandth star of the fifth magnitude per square degree. This value is as yet beyond the limits of accuracy of photometric measurements.

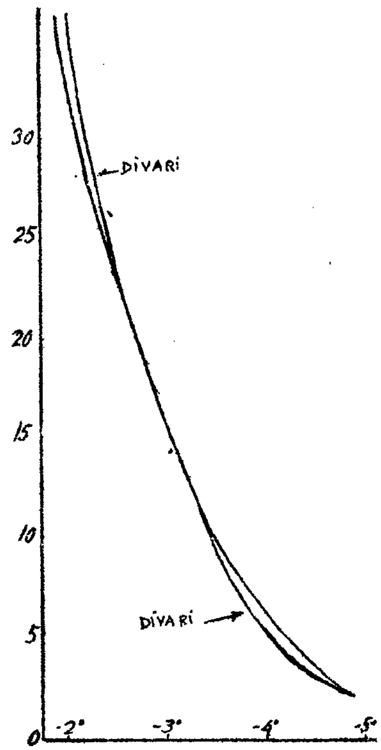

Fig. 4.

Thus, we may arrive at the conclusion that zodiacal twilight need not, despite the rapid increase in the brightness of zodiacal light with approach to the Sun, be taken into account.

The very extensive computational labors which made it possible to collate the data contained in Tables 1, 2 and 4, were performed in the Laboratory of Computer Mathematics of the Academy of Sciences Kazakh SSR under the direct guidance of Academician M. V. Pentkovskiy and with the participation of a researcher from the Astrophysical

Institute of that Academy, L. N. Tulenkova; to all concerned I wish to express my gratitude.

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